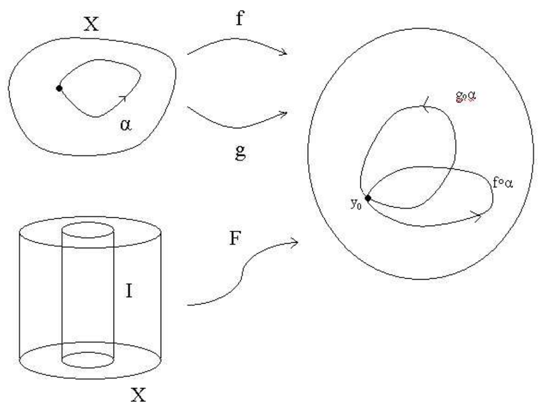
Homotopy invariance(preliminary version)

정리 1 *$f, g : (X, x_0) \to (Y, y_0)$ and $f \simeq g$ relative to x_0.*
$\Rightarrow f_\sharp = g_\sharp : \pi_1(X, x_0) \to \pi_1(Y, y_0)$.

증명 F(t,s)를 f 와 g 사이의 homotopy 라 하면, G(t,s)=F(α(t),s)는 $f \circ \alpha$ 와



$g \circ \alpha$ 사이의 homotopy를 준다. 따라서
$f_\sharp([\alpha]) = [f \circ \alpha] = [g \circ \alpha] = g_\sharp([\alpha])$. □

따름정리 2 *$f : (X, x_0) \to (Y, y_0), g : (Y, y_0) \to (X, x_0)$ and $g \circ f \simeq 1_X$ relative to x_0 , $f \circ g \simeq 1_Y$ relative to y_0.*
$\Rightarrow f_\sharp = (g_\sharp)^{-1} : \pi_1(X, x_0) \to \pi_1(Y, y_0)$ is an isomorphism.

증명 $g \circ f \simeq 1_X$ relative to x_0 이므로 $g_\sharp \circ f_\sharp = (g \circ f)_\sharp = (id_X)_\sharp = id$이다.

같은 방법으로 $f_\sharp \circ g_\sharp = (f \circ g)_\sharp = (id_Y)_\sharp = id$이다.

□

정의 1 X is homotopy equivalent to Y (or X has the same homotopy type as Y), denoted by $X \simeq Y$,
if $\exists f : X \to Y$ and $g : Y \to X$ such that $f \circ g \simeq 1_Y$ and $g \circ f \simeq 1_X$.
In this case f is called a homotopy equivalence.

Example. $\mathbf{R}^2 \setminus \{0\} \simeq S^1$.

$\mathbf{R}^2 \setminus \{0\}$를 X, S^1를 Y라고 하자.
$f(x) = \frac{x}{|x|}$, $g = inclusion$ 으로 주면 $f \circ g = 1_Y$ 인 것은 자명하다.

또한 $F(x,t) = (1-t)x + t\frac{x}{|x|}$ 이라고 두면

$$F(x,0) = x \qquad F(x,1) = \frac{x}{|x|} = f(x)$$

따라서 $g \circ f \simeq id_X$ 이고 X는 Y와 same homotopy type을 갖는다.

숙제 1.

1. $\mathbf{R}^n \setminus \{0\} \simeq S^{n-1}$.
2. $\simeq$ is an equivalence relation.
3. Möbius band 와 annulus는 homotopy type 이 같은가?
4. $T^2 \setminus \{point\} \simeq$ figure eight.