Foliation.

정의 1 *Let M be a n-dimensional $\mathcal{C}^\infty$ manifold. A k-dimensional submanifold $(N, incl)$ of M is called a* **foliation** *if*
(1) $M = N$ as a set.
(2) $\forall p \in N$, $\exists (U, x)$, a coordinate chart of M such that $x(U) = (-\epsilon, \epsilon)^n$ and the components of $N \cap U$ are the slices
$S_a = \{q \in U | (x^{k+1}(q), \cdots, x^n(q)) = (a_{k+1}, \cdots, a_n) = a\}, |a_j| < \epsilon, \ j = k+1, \cdot\cdot, n.$

Remark. $N \cap U$ is open in N : $N \cap U = i^{-1}(U)$ for $i : N \hookrightarrow M$, continuous. And each component of $N \cap U$ is open in N. Therefore S_a is a coordinate open neighborhood for N automatically and hence the topology of N is the coherent topology coming from $\{S_a\}$.

A connected component of a foliation N is called a leaf and N is partitioned by leaves.

따름정리 1 *Let $\mathcal{D}$ be a k-dimensional $\mathcal{C}^\infty$ involutive distribution on M. An integral manifold of $\mathcal{D}$ gives a foliation on M and each leaf is a maximal connected integral manifold of $\mathcal{D}$. Conversely, a foliation N on M gives rise to an involutive distribution given by $\mathcal{D}(p) = T_pN \subset T_pM$ whose integral manifold is the given foliation N.*

숙제 13. N is a foliation of $M \Rightarrow \exists$ an atlas of M such that each coordinate transition is given in the form

$$\begin{aligned} h : \mathbb{R}^k \times \mathbb{R}^{n-k} &\to \quad \mathbb{R}^k \times \mathbb{R}^{n-k} \\ (x, y) \quad &\mapsto (f(x, y), g(y)) \end{aligned}$$

Conversely such atlas gives a foliation structure on M.