

# Manifold with Boundary

**정의 1** A *topological manifold* of dimension  $n$  *with boundary* is a space  $M$  with the property that  $\forall p \in M, \exists$  a coordinate chart  $(U, \phi)$  of  $p$  s.t.  $\phi$  is a homeo onto an open set of  $(\mathbb{R}^n$  or)  $\mathbb{H}^n$ , where  $\mathbb{H}^n = \{x = (x_1, \dots, x_n) \in \mathbb{R}^n \mid x_1 \leq 0\}$

$M$ 의 boundary를  $\partial M$ 으로 쓴다.

즉,  $\partial M = \{p \in M \mid \phi(p) \in \{0\} \times \mathbb{R}^{n-1} =: \partial \mathbb{H}^n\}$

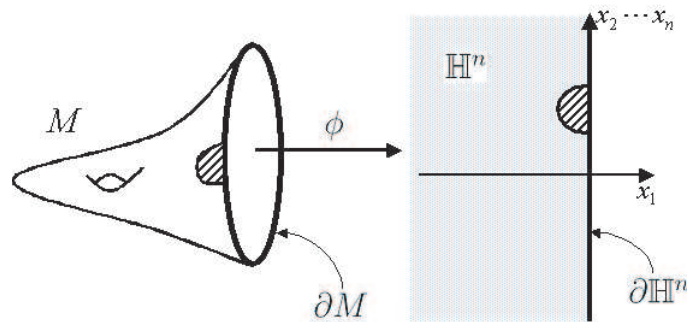


그림. Manifold with Boundary

**Remark** (invariance of domain)

In  $\mathbb{R}^n$ , 1-1 continuous image of open set is open.

위 Remark에 따르면, coordinate transition map은 interior point를 interior point로 boundary point는 boundary point로 보내므로,  $\partial M$ 은 coordinate chart의 선택에 관계없이 잘 정의된다.

A *smooth structure* on a manifold with boundary is defined exactly same as before.

(**note.** a map defined on an open subset of  $\mathbb{H}^n$  is smooth if it can be extended to a smooth map on an open set of  $\mathbb{R}^n$ )

- $M$  :  $n$ -dimensional  $C^\infty$  manifold with boundary  
 $\Rightarrow \partial M$  :  $(n - 1)$ -dimensional  $C^\infty$  manifold without boundary