* q- integers

On $\mathbb{C}_q[x,y] = \left\{ \sum_{m,n=0}^{\infty} a_{mn} x^m y^n \; ; \; a_{mn} \in \mathbb{C}(q) \text{ finite sum} \right\}$

give multiplication $\quad y \cdot x = q x \cdot y$

e.g. $\quad (1+y) \cdot x^2 = x^2 + y \cdot x^2 = x^2 + q xy \cdot x = x^2 + q^2 x^2 y$.

Lemma $\quad y^i \cdot x^j = q^{ij} \cdot x^j \cdot y^i$

Def $\quad (n)_q = 1 + q + \cdots + q^{n-1} = \dfrac{q^n - 1}{q - 1}$ $\qquad (0)_q^! = 1$

$(n)_q^! = (n)_q (n-1)_q \cdots (2)_q (1)_q$

$$\binom{m}{n}_q = \frac{(m)_q^!}{(n)_q^! \, (m-n)_q^!} \qquad (m \geq n) \qquad \binom{m}{n}_q = \binom{m}{m-n}_q$$

e.g.
$$\binom{4}{2}_q = \frac{(4)^!}{(2)^! \, (2)^!} = \frac{(4)_q (3)_q}{(2)_q (1)_q} = \frac{\frac{q^4-1}{q-1} \, \frac{q^3-1}{q-1}}{\frac{q^2-1}{q-1} \, \frac{q-1}{q-1}}$$

$$= (q^2+1)(q^2+q+1) = 1 + q + 2q^2 + q^3 + q^4$$

Prop [q- Pascal identity]

$$\binom{n}{k}_q = \binom{n-1}{k-1}_q + q^k \binom{n-1}{k}_q, = \binom{n-1}{k}_q + q^{n-k} \binom{n-1}{k-1}_q$$

Coro $\quad \binom{n}{k}_q$ is a <u>polynomial</u>.

Thm [q-binomial theorem]

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k}_q x^k y^{n-k}$$

Def [q-exponent]

$$\exp_q(z) := \sum_{n=0}^{\infty} \frac{1}{(n)_q^!} z^n$$

q-binomial thm

Prop $\quad yx = qxy \rightsquigarrow \exp_q(x+y) = \exp_q(x)\exp_q(y)$

Prop $\quad \{\exp_q(z)\}^{-1} = \sum_{n=0}^{\infty} (-1)^n q^{\frac{n(n-1)}{2}} \frac{z^n}{(n)_q^!} = \exp_{q^{-1}}(-z)$

$$\uparrow$$

$$\begin{cases} (n)_{q^{-1}} = (n)_q \cdot q^{-(n-1)} \\ (n)_{q^{-1}}^! = (n)_q^! \cdot q^{-\frac{n(n-1)}{2}} \\ \binom{m}{n}_{q^{-1}} = \binom{m}{n}_q \cdot q^{-n(m-n)} \end{cases}$$

* Modified q-integers.

$$[n] = \frac{q^{n/2} - q^{-n/2}}{q^{1/2} - q^{-1/2}} = \frac{q^{-n/2}}{q^{-1/2}} \frac{(q^n - 1)}{q-1} = q^{-\frac{n-1}{2}} (n)_q .$$

So $\quad [n]^! = [n] \cdots [2][1] = q^{-\frac{0+1+\cdots+(n-1)}{2}} (n)_q^! = q^{-\frac{n(n-1)}{4}} (n)_q^!$

$$\begin{bmatrix} m \\ n \end{bmatrix} = \frac{[m]^!}{[n]^![m-n]^!} = q^{-\frac{m(m-1)}{4} + \frac{n(n-1)+(m-n)(m-n-1)}{4}} \binom{m}{n}_q$$

$$= q^{\frac{n(n-m)}{2}} \binom{m}{n}_q .$$

So, $\displaystyle \exp_q(z) = \sum_{n=0}^{\infty} \frac{q^{\frac{n(n-1)}{4}}}{[n]!} z^n$, obtained.

Prop $\displaystyle R = q^{\frac{H\otimes H}{4}} \exp_q\left((q^{\frac{1}{2}} - q^{-\frac{1}{2}}) (E \otimes F) \right) \in U_q \hat{\otimes} U_q$

is an Universal R-matrix, i.e, (U_q, R) is a quasi-triangular Hopf algebra.

<pf> We check

$R = \sum \alpha_i \otimes \beta_i$

$\downarrow$

$$\begin{cases} R_{13} = \sum \alpha_i \otimes 1 \otimes \beta_i \\ R_{12} = \sum \alpha_i \otimes \beta_i \otimes 1 \\ R_{23} = \sum 1 \otimes \alpha_i \otimes \beta_i \end{cases}$$

① $(P \circ \Delta)(x) = R \Delta(x) R^{-1}$

② $(\Delta \otimes id) R = R_{13} R_{23}$

③ $(id \otimes \Delta) R = R_{13} R_{12}$

② 左 $= (\Delta \otimes id)\left(q^{\frac{H\otimes H}{4}} \exp_q\left((q^{\frac{1}{2}} - q^{-\frac{1}{2}}) E \otimes F \right) \right)$

$$= q^{\frac{(H\otimes 1 + 1\otimes H)\otimes H}{4}} \exp_q\left((q^{\frac{1}{2}} - q^{-\frac{1}{2}}) (E \otimes K + 1 \otimes E) \otimes F \right)$$

右 $= \underbrace{q^{\frac{H\otimes 1\otimes H}{4}} \exp_q\left((q^{\frac{1}{2}} - q^{-\frac{1}{2}}) E \otimes 1 \otimes F \right)}_{R_{13}} \cdot \underbrace{q^{\frac{1\otimes H\otimes H}{4}} \exp_q\left((q^{\frac{1}{2}} - q^{-\frac{1}{2}}) 1 \otimes E \otimes F \right)}_{R_{23}}$

We calculate $\exp_q\left((q^{\frac{1}{2}}-q^{-\frac{1}{2}}) E\otimes 1\otimes F\right)\cdot q^{\frac{1\otimes H\otimes H}{4}} = ✪$

Note that $(E\otimes 1\otimes F)\cdot q^{\frac{1\otimes H\otimes H}{4}} = q^{\frac{1\otimes H\otimes (H+2)}{4}} (E\otimes 1\otimes F)$

$$\boxed{\begin{aligned} F\cdot q^{\frac{H}{4}} &= q^{\frac{H+2}{4}}\cdot F \\ \| \qquad & \qquad \| \\ F\cdot \sum \frac{(\hbar)^n}{n!}\frac{H^n}{4^n} & \end{aligned}}$$

$$= q^{\frac{1\otimes H\otimes H}{4}}\cdot q^{1\otimes \frac{H}{2}\otimes 1}(E\otimes 1\otimes F)$$

$$\cancel{= q^{\frac{1\otimes H\otimes H}{4}} (1\otimes K\otimes 1)(E\otimes 1\otimes F)}$$

$$= q^{\frac{1\otimes H\otimes H}{4}} (E\otimes K\otimes F).$$

We can't have $q^{\frac{1\otimes H\otimes H}{4}} = q^{1\otimes K\otimes K}$.

So $✪ = q^{\frac{1\otimes H\otimes H}{4}}\cdot \exp_q\left((q^{\frac{1}{2}}-q^{-\frac{1}{2}}) E\otimes K\otimes F\right)$.

Now it remains to show

$$\exp_q\left((q^{\frac{1}{2}}-q^{-\frac{1}{2}}) (E\otimes K\otimes F)\right)\cdot \exp_q\left((q^{\frac{1}{2}}-q^{-\frac{1}{2}}) (1\otimes E\otimes F)\right)$$

$$= \exp_q\left((q^{\frac{1}{2}}-q^{-\frac{1}{2}}) (E\otimes K\otimes F + 1\otimes E\otimes F)\right)$$

which is clear from the q-exponent law and

$$(E\otimes K\otimes F)\cdot(1\otimes E\otimes F) = E\otimes KE\otimes F^2 = E\otimes qEK\otimes F^2$$

$$= q\cdot(1\otimes E\otimes F)(E\otimes K\otimes F)$$

③ is proved similarly.

Now we show $(P \circ \Delta)(X) = R\Delta(X)R^{-1}$ for generators $K^{\pm}, E, F$.

Then we will be done.

First, we can check that R is invertible. $\rightarrow$ w/ inverse $\exp_{q^{-1}}(-x)$

$(\because q^{\square}, \exp_q \square$ are invertibles$)$

Ⓐ $(P(K^{\pm} \otimes K^{\pm})) \cdot R \overset{?}{=} R \cdot (K^{\pm} \otimes K^{\pm})$

$$\begin{array}{c} \| \\ (K^{\pm} \otimes K^{\pm}) \cdot R \end{array}$$

左 $= (K \otimes K)\, q^{H \otimes H/4} \exp_q((q^{\frac{1}{2}} - q^{-\frac{1}{2}}) E \otimes F) = q^{H \otimes H/4} (K \otimes K) \exp_q((q^{\frac{1}{2}} - q^{-\frac{1}{2}})(E \otimes F))$ $\overset{\|}{\text{右}}$

(Note that $(K \otimes K)(E \otimes F)^n = (E \otimes F)^n (K \otimes K)$.)

Ⓑ
$$\begin{cases} (P(E \otimes K + 1 \otimes E)) R \overset{?}{=} R(E \otimes K + 1 \otimes E) \\ (P(K^{-1} \otimes F + F \otimes 1)) R \overset{?}{=} R(F \otimes 1 + K^{-1} \otimes F) \end{cases}$$

only check the 1st relation.

$$\begin{aligned} \text{左} &= \underline{(K \otimes E + E \otimes 1)\, q^{H \otimes H/4}} \exp_q((q^{\frac{1}{2}} - q^{-\frac{1}{2}}) E \otimes F) \\ &= \left(q^{H \otimes (H-2)/4} K \otimes E + q^{(H-2) \otimes H/4} E \otimes 1\right) \cdot \exp_q\left((q^{\frac{1}{2}} - q^{-\frac{1}{2}}) E \otimes F\right) \\ &= q^{H \otimes H/4} \underline{(1 \otimes E + E \otimes K^{-1}) \cdot \exp_q\left((q^{\frac{1}{2}} - q^{-\frac{1}{2}}) E \otimes F\right)} \end{aligned}$$

$$(1\otimes E + E\otimes K^{-1})(E\otimes F)^n = E^n\otimes EF^n + E^{n+1}\otimes K^{-1}F^n$$

Claim $EF^n = F^nE + \dfrac{[n]\cdot q^{-(n-1)/2}}{q^{1/2}-q^{-1/2}}\left(F^{n-1}K - K^{-1}F^{n-1}\right)$.

⟨pf⟩ Induction on n.

$$EF^{n+1} = \left(F^nE + \frac{[n]\cdot q^{-(n-1)/2}}{q^{1/2}-q^{-1/2}}\left(F^{n-1}K - K^{-1}F^{n-1}\right)\right)F$$

$$= F^n\left(FE + \frac{K-K^{-1}}{q^{1/2}-q^{-1/2}}\right) + \frac{[n]\cdot q^{-(n-1)/2}}{q^{1/2}-q^{-1/2}}\left(F^{n-1}\underbrace{KF}_{q^{-1}FK} - K^{-1}F^n\right)$$

$$= F^{n+1}E + \frac{1+[n]q^{-(n-1)/2}\cdot q^{-1}}{q^{1/2}-q^{-1/2}}F^nK - \frac{q^{-n}+[n]\cdot q^{-(n-1)/2}}{q^{1/2}-q^{-1/2}}K^{-1}F^n$$

$$= F^{n+1}E + \frac{[n+1]q^{-n/2}}{q^{1/2}-q^{-1/2}}\left(F^nK - K^{-1}F^n\right).$$

So, 左 $= q^{H\otimes H/4}\sum \dfrac{q^{n(n-1)/4}}{[n]!}(q^{1/2}-q^{-1/2})^n\cdot E^n\otimes\left(F^nE + \dfrac{[n]\cdot q^{-(n-1)/2}}{q^{1/2}-q^{-1/2}}\left(F^{n-1}K\cancel{-K^{-1}F^{n-1}}\right)\right)$

$+\, q^{H\otimes H/4}\sum \cancel{\dfrac{q^{n(n-1)/4}}{[n]!}(q^{1/2}-q^{-1/2})^n}\, E^{n+1}\otimes K^{-1}F^n$

shift

$= q^{H\otimes H/4}\sum \dfrac{q^{n(n-1)/4}}{[n]!}(q^{1/2}-q^{-1/2})^n\, \cancel{E^n\otimes(F^nE+F^nK)}$
$(E^n\otimes F^nE + E^{n+1}\otimes F^nK)$

$= q^{H\otimes H/4}\ \exp_q\left((q^{\frac{1}{2}}-q^{-\frac{1}{2}})(E\otimes F)\right)(1\otimes E + E\otimes K)$

$=$ 右.

* For $R = \sum \alpha_i \otimes \beta_i$, we've defined $u = \sum S(\beta_i)\alpha_i \in \hat{U}$

By calculation, we have

$$u = q^{-H^2/4} \sum_{n=0}^{\infty} \frac{q^{3n(n-1)/4}}{[n]!} (q^{-1/2} - q^{1/2})^n F^n K^{-n} E^n \to \boxed{19}-1$$

And for

$$v = q^{-H^2/4} \sum_{n=0}^{\infty} \frac{q^{n(3n+1)/4}}{[n]!} (q^{-\frac{1}{2}} - q^{\frac{1}{2}})^n F^n K^{-n-1} E^n, \to \boxed{19}-2$$

we have the conditions of Ribbon Hopf alg, i.e.

⓪ v is central

① $v^2 = S(u)u$

② $\Delta(v) = (v \otimes v) \cdot (R_{21} R)^{-1}$

③ $S(v) = v$

④ $\varepsilon(v) = 1$

$KF = q^{-1/2} FK$

$(-KF)^n = (-1)^n \cdot KFKF \cdots KF$
$= (-1)^n \cdot q^{-1/2 - 2/2 - \cdots - n/2} F^n K^n$

$\uparrow$

$S(F) = -KF$

$\boxed{Z}$ $u = \sum S(\beta_i)\alpha_i$ if $R = \sum \alpha_i \otimes \beta_i$

$$= q^{-H \otimes H/4} \cdot \sum \frac{q^{\frac{n(n-1)}{4}}}{[n]!} (q^{\frac{1}{2}} - q^{-\frac{1}{2}})^n \cdot (S(F))^n E^n$$

$$= q^{-H^2/4} \sum \frac{q^{\frac{n(n-1)}{4}}}{[n]!} (q^{-\frac{1}{2}} - q^{\frac{1}{2}})^n \cdot q^{-\frac{n(n+1)}{4}}$$

* Recall $q^{H\otimes H/4} = 1 + \frac{\hbar}{4}(H\otimes H) + \frac{\hbar^2}{2!\cdot 4^2}(H^2\otimes H^2) + \cdots = \sum a_i H^i \otimes H^i$

$$\rightsquigarrow R = \sum_{i,n} \frac{q^{n(n-1)/4}}{[n]!}(q^{1/2}-q^{-1/2})^n a_i \cdot H^i E^n \otimes H^i F^n .$$

$$\rightsquigarrow u = \sum_{i,n} \frac{q^{n(n-1)/4}}{[n]!}(q^{1/2}-q^{-1/2})^n a_i \cdot S(E^n)\, S(H^i)\, H^i F^n$$

Note $\sum_i a_i S(H^i) H^i = \sum a_i \cdot (-H)^i H^i = 1 - \frac{\hbar}{4} H^2 + \frac{\hbar^2}{2!\,4^2} H^4 - \cdots$

$$= q^{-H^2/4} .$$

$$\therefore \quad u = \sum_n \frac{q^{n(n-1)/4}}{[n]!}(q^{1/2}-q^{-1/2})^n \cdot (-KF)^n \cdot q^{-\frac{H^2}{4}} E^n$$

(Recall $\cancel{KF\cdot q^{-\frac{H^2}{4}} = Kq^{-\frac{(H-2)^2}{4}}\cdot F = q^{-\frac{H^2}{4}+\frac{3}{4}H-1}\cdot F}$

$$KF\cdot q^{-\frac{H^2}{4}} = q^{\frac{H}{2}} F q^{-\frac{H^2}{4}} = q^{\frac{H}{2}}\cdot q^{-\frac{(H+2)^2}{4}} F = q^{-\frac{H^2}{4}}\cdot q^{-\frac{H}{2}}\cdot q^{-1} F$$

$$= q^{-1}\cdot q^{-\frac{H^2}{4}} F\cdot q^{-\frac{H-2}{2}} = q^{-\frac{H^2}{4}}\cdot F\cdot K^{-1}$$)

$(FK^{-1})^n = q^{\frac{n(n-1)}{2}} F^n K^{-n}$

$$\therefore \quad u = \sum_n \frac{q^{n(n-1)/4}}{[n]!}(q^{-\frac{1}{2}}-q^{\frac{1}{2}})^n \cdot q^{-\frac{H^2}{4}} (FK^{-1})^n E^n$$

$$= q^{-\frac{H^2}{4}} \sum_n \frac{q^{n(n-1)/4}}{[n]!}(q^{-\frac{1}{2}}-q^{\frac{1}{2}})^n \cdot q^{\frac{n(n-1)}{2}} F^n K^{-n} E^n$$

$$= q^{-\frac{H^2}{4}} \sum_n \frac{q^{3n(n-1)/4}}{[n]!}(q^{-\frac{1}{2}}-q^{\frac{1}{2}})^n F^n K^{-n} E^n .$$

$\boxed{\text{Lemm}}$ $S(u) = uK^{-2}$.

<pf> By tedious calculation!

Hence $uS(u) = u^2K^{-1}$ and u has weight 0.

So we chose $v = uK^{-1}$.

$KE^nK^{-1} = (KEK^{-1})^n = q^{\frac{n}{2}}$

$$v = q^{-\frac{H^2}{4}} \sum \frac{q^{3n(n-1)/4}}{[n]!}\left(q^{-\frac12}-q^{\frac12}\right)^n F^n K^{-n} E^n \cdot K^{-1}$$

$$= q^{-\frac{H^2}{4}} \sum \frac{q^{n(3n+1)/4}}{[n]!}\left(q^{-\frac12}-q^{\frac12}\right)^n F^n K^{-n-1} E^n .$$

* v action on V_m.

Only calculate $q^{-\frac{H^2}{4}}\, q^{1\cdot 4/4}\left(q^{-\frac12}-q^{\frac12}\right)^1 F^0 . K^{-1} . w_0$ term. ($\because$ v is central.)

$$\cancel{= q^1 \cdot q^{-\frac{H^2}{4}} \cdot \underbrace{H}_{q^{-1/2}-q^{1/2}} \cdot \left(q^{\frac{m}{2}}\right)^{-1} \cdot w_0 = q^{1-\frac{m}{2}} \cdot q^{-\frac{m^2}{4}} w_0 = q^{-\ \ 4}}$$

$$\cancel{= \left(q^{\frac12}-q^{\frac32}\right) q^{-\frac{m}{2}-\frac{m^2}{4}} w_0}$$

$$\cancel{\therefore\ v = \left(q^{\frac{2-2m-m^2}{4}} - q^{\frac{6-2m-m^2}{4}}\right) Id_{V_m}}$$

Or $uK^{-1}.w_0 = q^{-\frac{M}{2}}\, u\cdot w_0 = q^{-\frac12}\, q^{H^2/4}\left(q^{\frac12}-q^{-\frac12}\right).w_0$

$= (1-q^{-1})\cdot q^{\frac{H^2}{4}} w_0 = q^{\frac{m^2}{2}}(1-q^{-1}) w_0$.

* The R-matrix on $V = V_2^q = V_q(1) = \langle w_0, w_1 \rangle$ given by the universal R-matrix.

$$\begin{cases} E \cdot w_0 = 0, & E \cdot w_1 = w_0 \\ K \cdot w_0 = q^{1/2} w_0, & K \cdot w_1 = q^{-1/2} w_1 \\ F \cdot w_0 = w_1, & F \cdot w_1 = 0 \end{cases}$$

Recall the universal R-matrix

$$R = q^{H\otimes H/4} \cdot \exp_q\left((q^{\frac{1}{2}} - q^{-\frac{1}{2}}) E \otimes F \right).$$

Also note that $E^2 = F^2 = 0$ on $V = V_2^q$.

Hence we have (4.30)

$$R = p_0 q^{H\otimes H/4} \underbrace{\left(1\otimes 1 + (q^{\frac{1}{2}} - q^{-\frac{1}{2}}) E\otimes F \right)}_{T} \text{ on } V\otimes V.$$

$$\begin{cases}
w_0 \otimes w_0 \xmapsto{T} w_0\otimes w_0 + 0 \xmapsto{q^{H\otimes H/4}} q^{\frac{1}{2}\cdot\frac{1}{2}} w_0\otimes w_0 \\
w_1 \otimes w_0 \xmapsto{T} w_1\otimes w_0 + (q^{1/2} - q^{-1/2}) w_0 \otimes w_1 \longmapsto q^{-\frac{1}{2}\cdot\frac{1}{2}} w_1 \otimes w_0 + (q^{\frac{1}{2}} - q^{-\frac{1}{2}}) q^{\frac{1}{2}\cdot(-\frac{1}{2})} w_0 \otimes w_1 \\
w_0 \otimes w_1 \xmapsto{T} w_0 \otimes w_1 + 0 \longmapsto q^{\frac{1}{2}\cdot(-\frac{1}{2})} w_0 \otimes w_1 \\
w_1 \otimes w_1 \xmapsto{T} w_1 \otimes w_1 + 0 \longmapsto q^{-\frac{1}{2}\cdot(-\frac{1}{2})} w_1 \otimes w_1
\end{cases}$$

Hence R has matrix form

$$p_0 \left(\begin{array}{c|cc|c} q^{\frac{1}{4}} & & & \\ \hline & q^{-\frac{1}{4}} & 0 & \\ & q^{\frac{1}{4}} - q^{-\frac{3}{4}} & q^{-\frac{1}{4}} & \\ \hline & & & q^{\frac{1}{4}} \end{array} \right)$$

$$= \left(\begin{array}{c|cc|c} q^{1/4} & & & \\ \hline & 0 & q^{-1/4} & \\ & q^{-1/4} & q^{1/4} - q^{-3/4} & \\ \hline & & & q^{1/4} \end{array} \right)$$

* Check $q^{1/4}R - q^{-1/4}R^{-1} = (q^{1/2}-q^{-1/2})\, id_{V\otimes V}$.

<pf>

$$\det(R) = q^{\frac14}\cdot q^{\frac14}\cdot\left(-q^{-\frac14}\cdot q^{-\frac14}\right) = -1 .$$

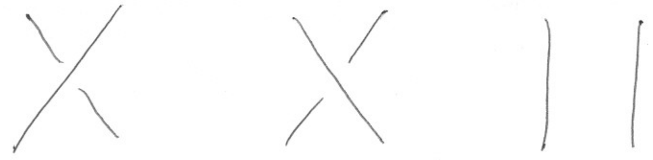
$$\therefore\ R^{-1} = -\begin{pmatrix} q^{\frac14}\cdot(-q^{-\frac12}) & & & \\ & q^{\frac12}(q^{1/4}-q^{-3/4}) & -q^{\frac14} & \\ & -q^{\frac14} & 0 & \\ & & & q^{\frac14}\cdot(-q^{-1/2}) \end{pmatrix}$$

$$= \begin{pmatrix} q^{-\frac14} & & & \\ & -q^{\frac34}+q^{-\frac14} & q^{\frac14} & \\ & q^{\frac14} & 0 & \\ & & & q^{-\frac14} \end{pmatrix}$$

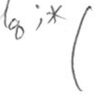
$$\therefore\ 左 = \begin{pmatrix} q^{\frac12}-q^{-\frac12} & & & \\ & -(-q^{\frac12}+q^{-\frac12}) & 1-1 & \\ & (q^{\frac12}-\cancel{\ \ }) - q^{\frac12} & (q^{\frac12}-q^{-\frac12})-0 & \\ & & & q^{\frac12}-q^{-\frac12} \end{pmatrix}$$

$$= \left(q^{\frac12}-q^{-\frac12}\right)\begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}.$$

* Let



L_+ $\qquad$ L_- $\qquad$ L_0

Recall $Q^{\mathcal{U}_q;*}($  $) =$  , $\quad Q^{\mathcal{U}_q;*}($  $) =$ 

And $V \simeq V^*$ as $\mathcal{U}_q$-module (by highest wt theory)

$\boxed{\text{Prop}}$ $q^{1/4} Q^{\mathcal{U}_q;*}(L_+) - q^{-1/4} Q^{\mathcal{U}_q;*}(L_-) = (q^{1/2} - q^{-1/2}) Q^{\mathcal{U}_q;*}(L_0)$

* Recall $Q^{\mathcal{U}_q;*}(\bigcirc) =$  $= vu^{-1} = u^{-1}v = \underline{K^{-1}}$

* Now for a good element h and $\mathcal{F}_n : B_n \to \mathrm{End}(V^{\otimes n})$,

the trace $b \mapsto \mathrm{tr}(h^{\otimes n} \cdot \mathcal{F}_n(b))$ gives a <u>Link invariant</u>.

(Jones polynomial)

Normalization of R by $\rho(v) = q^{-\frac{3}{4}} \cdot \mathrm{id}_V$

$\left(\because \underbrace{q^{-\frac{H^2}{4}}(K^{-1} + q(q^{-\frac{1}{2}} - q^{\frac{1}{2}})FK^{-2}E)}_{v \text{ on } V_2^q} \cdot w_0 = q^{-\frac{H^2}{4}} q^{-\frac{1}{2}} w_0 = q^{-\frac{1}{2} - \frac{1}{4}} w_0 \right)$

In general, on V_n we have $q^{\frac{1-n^2}{4}}$.

$\rightsquigarrow \begin{pmatrix} q^{-1/2} & & & \\ & 0 & q^{-1/2} & \\ & q^{-1/2} & q^{-1/4} - q^{-3/4} & \\ & & & q^{-1/2} \end{pmatrix}$ gives <u>unframed</u> link invariants.